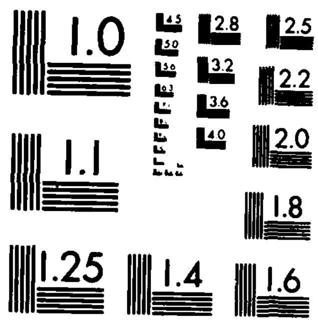
AD-A128 553 AN EFFECT OF TURBULENT DIFFUSION ON THE GLOW DISCHARGE-TO-ARC TRANSITION(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA R J WALLACE MAR 83 1/1

UNCLASSIFIED F/G 20/4 NL

END
DATE
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

2

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

AN EFFECT OF TURBULENT DIFFUSION ON THE GLOW DISCHARGE-TO-ARC TRANSITION

by

Richard J. Wallace

March 1983

Thesis Advisor: O. Biblarz

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
An Effect of Turbulent Diffusion on the Glow Discharge-to-Arc Transition

5. TYPE OF REPORT & PERIOD COVERED
Master's Thesis
March 1983

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
Richard J. Wallace

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Naval Postgraduate School
Monterey, California 93940

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
Naval Postgraduate School
Monterey, California 93940

12. REPORT DATE
March 1983

13. NUMBER OF PAGES
74

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
glow-to-arc-transition
turbulent diffusion
laminar diffusion
glow-discharge
streamer
breakdown
cathode glow

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
In this thesis, an effect of turbulent gas flow on the glow discharge-to-arc transition is investigated. This is accomplished by solving the continuity equation for charged particles by two different numerical processes. They are a linearized solution and a non-linear numerical solution. The numerical results using an axi-symmetric cylindrical geometry show the charge particle density profile with respect to radial distance, time and diffusion coefficient.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

20. ABSTRACT (Continued)

In nitrogen at a time equal to $4\tau_r$, the radius of the streamer in near turbulent conditions (β=50) was found to be 2.5 times larger than in laminar conditions ($\beta=10^4$). τ_r is the characteristic recombination time and β is the ratio of the characteristic diffusion time divided by τ_r.

The effective increase in the diffusion coefficient between laminar and turbulent gas flow causes the charge density profile to expand more radially outward. This reduces the charge density within the original streamer volume tube causing a reduction in the conductivity of the streamer. This reduction in conductivity may possibly lead to a delay in the development of a breakdown streamer. As a result, the gas may be raised to higher current levels before breakdown occurs.

An Effect of Turbulent Diffusion on the Glow Discharge-to-Arc Transition

by

Richard J. Wallace
Lieutenant, United States Navy
B.A., Hamilton College, 1975

Submitted in partial fulfillment of the requirements
for the degrees of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

and

AERONAUTICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
March 1983

Author: Richard J. Wallace

Approved by: Oscar Biblarz
Thesis Advisor

Fred Schwirzke
Second Reader

Donald M. Layton
Chairman, Department of Aeronautics

J. M. Dyer
Dean of Science and Engineering

ABSTRACT

In this thesis, an effect of turbulent gas flow on the glow discharge-to-arc transition is investigated. This is accomplished by solving the continuity equation for charged particles by two different numerical processes. They are a linearized solution and a non-linear numerical solution. The numerical results using an axi-symmetric cylindrical geometry show the charge particle density profile with respect to radial distance, time and diffusion coefficient.

In nitrogen at a time equal to $4\tau_r$, the radius of the streamer in near turbulent conditions ($\beta = 50$) was found to be 2.5 times larger than in laminar conditions ($\beta = 10^4$). τ_r is the characteristic recombination time and β is the ratio of the characteristic diffusion time divided by τ_r.

The effective increase in the diffusion coefficient between laminar and turbulent gas flow causes the charge density profile to expand more radially outward. This reduces the charge density within the original streamer volume tube causing a reduction in the conductivity of the streamer. This reduction in conductivity may possibly lead to a delay in the development of a breakdown streamer. As a result, the gas may be raised to higher current levels before breakdown occurs.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

α = General recombination coefficient

α_2 = Two-body recombination coefficient

α_3 = Three-body recombination coefficient

β = Diffusion time/recombination time

D = Diffusion Coefficient

D_a = Ambipolar diffusion coefficient

n = Charged particle density

n_o = Initial charged particle density

n_e = Electron density

n_i = Ion density

r_o = Initial streamer "tube" radius

τ = General time parameter

τ_r = Characteristic recombination time

τ_D = Characteristic diffusion time

τ_{ei} = Electron and (electron/ion) collision time

U = Gas flow velocity

x_o = 2-dimensional streamer radius

ACKNOWLEDGEMENTS

I would like to publicly thank my wife, Vicci, for sharing me with this thesis, Dr. O. Biblarz for keeping my feet on the ground, Dr. R. Ball for assistance in the numerical analysis, and the United States Navy for permitting me to obtain a graduate education.

I. INTRODUCTION

The belief that a turbulent flow of an electrically excited lasing medium leads to an increase in the glow-discharge stability has been demonstrated for a number of years [Ref. 1]. However, few experiments that yield definitive results have been reported. The increase in glow discharge stability has been attributed to the dramatic increase in the diffusion transport mechanism and thermal conductivity of the gas between laminar and turbulent flow. This may rapidly dilute the local charge carrier density in the gas and thus delay the onset of the streamer-to-arc transition. Previous experimental and theoretical analyses have centered around the prevention of the breakdown discharge; that is the glow-discharge-to-arc transition [Ref. 2]. This is done by observing the current-to-voltage relationship of a gas with and without turbulent flow. These investigations generally do not attempt to answer the question of what is the effect of turbulent flow on the onset of the arc. This work proposes to analyze the events that reduce the charge carrier density in the arc channel. With this in mind, a model is presented which describes the onset and delay mechanisms involved in the turbulent flow, glow-discharge-to-arc transition.

Even though we pretend to study and model the effects of turbulent flow on the glow-discharge-to-arc transition in an electrically pumped laser, the information provided should be applicable to the onset of many spark or arc situations.

The narrative is written at a level where a graduate student with fundamental skills in mechanics and an introduction to basic atomic processes will be able to understand the subtleties of discharges. With such a limited background in mind, a physical model of the ionization and sparking process is described. The process of ionization and de-ionization are also reviewed.

The presentation will begin with the modern understanding of ionization in a gas. Then, theories which pertain to the development of the transition to an arc will be discussed and experimental results pertaining to the turbulent flow arc transition will be introduced. Afterwards, an analysis of the streamer decay will be developed. Finally, the results of a numerical analysis will be used to substantiate the delay mechanism theorized with the glow-discharge-to-arc transition in a turbulent flow situation.

The following is a description of an arc in a gaseous medium. It relates the voltage-current relationship in a gas and tells in a qualitative manner how an arc occurs and its relation to the glow-discharge.

> "In the typical voltage-current characteristic of a gas the last region of discharge, that is, the one having the highest current, is called the 'arc'. In this region, the current is about two orders of magnitude higher than in the glow, whereas the voltage is roughly an order of magnitude lower.
>
> The arc is probably the form of the discharge least understood theoretically, although it has been used for a long time in a number of applications.
>
> The glow-to-arc transition occurs when the abnormal glow has acquired a critical value of the voltage and when the power supply has an adequately low internal resistance capable of sustaining the high currents required for the arc.

Arcs have been known since the early days of electricity because of their immense light intensity, which has been and still is used for lighting purposes. Because of the arc's high temperature, extensive research has been recently conducted to produce the temperatures needed to trigger thermonuclear fusion.

In the arc the current density is much higher than the glow. The increased light intensity makes it very difficult to recognize the various regions within the arc. At atmospheric pressure in air, the arc seems to have a very bright core with less bright surroundings which resemble a hot flame. The core itself has no discernable structure. An arc keeps changing position because of the continuous heating that leads to convection of the hot gas upward. A horizontal arc will therefore have an arched shape, from which it derived it's name.

The arc is a very hot plasma. Usually, arcs are produced in the near atmospheric range and therefore the charged-particle density in their cores will range from 10^{14} to 10^{18} electrons/cm^{3}." [Ref. 3].

Many familiar discharge events surround us every day. The light emitted from a fluorescent lamp is a glow-discharge type event which occurs at a reduced pressure. Lightning is a large scale electric discharge at atmospheric pressure. Glow discharges are often used to excite electrons to high energy states in electrically pumped gas lasers. A breakdown or arc within the laser can cause structural damage. More importantly, arcing destroys the population inversion needed to achieve lasing action. To obtain the maximum power output from this type of laser requires operating the glow discharge 'pump' very near arcing conditions. With the addition of turbulent flow of the gas, the transition from glow discharge to arc is delayed. This permits either a greater margin of safety in laser operation or higher power levels produced by the laser.

II. METHODS OF IONIZATION

"The process of liberating an electron from a gas with either the simultaneous production of a positive ion or with the increase of positive ion charge is called ionization" [Ref. 4].

The two most important ionization processes in a gas are by electron collision and by the absorption of radiation (photoionization). If ionization if a neutral atom is to uccur by collision directly, the impinging electron must have a kinetic equal to or greater than the ionization energy of the particular atom. Atoms may also be ionized in steps by electron collisions with energy less than that needed for direct ionization. This is a less efficient method of ionization, however.

H. Hertz found that a spark between two electrodes at a lower voltage could be created when the light emission from another spark were permitted to impinge on it. He showed that the ultraviolet light from the first spark striking the cathode of the second caused the breakdown. The ionization energy of hydrogen, for example, is about 14 electron volts. For a photon to ionize hydrogen directly requires that it have a frequency of about $3.4 \cdot 10^{15}$ Hz, which is in the near ultraviolet. This assumes one-hundred percent efficiency and direct liberation of an electron due to the photon absorption by the hydrogen atom.

When electrodes are put into a gas such as air or nitrogen and a weak voltage is applied to their terminals, a very weak current, much less than a picoampere by several orders of magnitude will appear. This

current will increase with voltage until all of the charge carriers naturally present are being used and current saturation occurs. This means that all of the charge carriers move to the electrodes with little loss due to recombination and diffusion to the walls. These charge carriers are created naturally by external radiation such as cosmic rays. Further increases in the applied voltage beyond the plateau discussed above will yield a rapidly increasing current flow on the order of a picoampere. This zone is called the Townsend discharge region and the new charge carriers are being created by ionizing collisions between fast moving electrons and neutral atoms. These fast moving electrons are accelerated by the electric field and upon colliding with a neutral gas particles may either excite the gas or ionize it, further increasing the density of charged particles. This ionization will take place when the kinetic energy gained by the electron between collisions is greater than that required for ionization. Clearly, the lower the gas density, the farther the electron will travel between collisions (lower collision frequency) and the larger will be it's average kinetic energy.

In this scheme, the total number of available electrons will increase if the initial electrons have the energy required to ionize the gas and a self-sustaining current will develop. The new electrons will gain energy in the same manner and the number of electrons will increase between the cathode and the anode. This process is called an electron avalanche due to the rapid increase in electron production.

After this interval, the current increases by orders of magnitude with a slight increase of applied voltage. Here, the current is independent of external ionizing sources and the current becomes self-sustaining. Further increases in the current flow by reducing the external limiting resistor produces a glow in the gas. This is the primary region of interest in an electrically pumped laser.

Glow-discharge is one of the oldest known discharge phenomena. It develops very easily when the gas pressure is less than one-hundred Torr. Glow-discharge will also appear under special conditions at higher pressures. The diffuse glow-discharge region is well suited as a laser pumping scheme. This is primarily due to the relatively good coupling of electron energy to excited atomic states. These states can be either vibrational or electronic in nature.

When electrons leave the cathode, they are accelerated by the applied electric field. As the electrons are accelerated, they quickly become too energetic to recombine with passing ions. As the electrons' energy increase, they ionize neutral atoms by collision. This process retards the motion of the electrons enough that they are now able to recombine with passing ions. The result of this two-body recombination is the emission of a photon which carries off excess energy in the form of light. In the glow-discharge this region is known as the negative glow.

Finally, the transition to an arc discharge occurs when the current is allowed to increase further. A highly ionized channel forms which collapses the glow. The current through this channel is generally limited only by the external circuit network. The arc breakdown of the

gas is an irreversible transition. If this plasma channel is short lived and at atmospheric pressures, it is known as a spark. If the current is steady, the degree of ionization is determined by the magnitude of the current and a plasma state known as an arc is developed.

There are only two generally accepted breakdown theories. The first was developed by Townsend in the early 1900s. In this method, successive electron avalanches between the cathode and anode develop in which every avalanche produces at least one following avalanche. A highly ionized channel is produced in which the conductivity becomes so high that the current through it is limited by the external circuit elements. This theory remained unchallenged until the development of high speed oscilloscopes. With these improved observation tools, it was found that the development times of the ionized channel was very much faster than the Townsend theory could predict. The period of time required for the irreversible transition is known as the formative lag of breakdown. Formative time lags on the order of ten nanoseconds were first recorded in the 1920s. These short times almost totally precluded the use of positive ion schemes in breakdowns due to their relatively large mass and limited the role of the electron avalanches.

The ionization channels were seen to be heavily branched and filamentary in nature. They could be started at low voltages at asymmetric nodes and could reach large distances. This channel is also known as a streamer. If breakdown occurs by a transition from a streamer to a spark, the mechanism involved is often called a streamer-breakdown. The streamers develop in time intervals on the order of ten nanoseconds [Ref. 5].

The second method which causes breakdown to occur is based on photons which ionize the gas. During the buildup of the primary avalanche, excitation of the gas as well as ionization takes place. These excited states have lifetimes on the order of picoseconds. Thus, before the avalanche can reach its full size, photons will be emitted from these excited states as they return to the ground state.

The photons will be emitted isotropically and will be absorbed at various distances from their origin, depending on the absorption coefficient of the gas. Numerous processes then take place which lead to photoionization. Newly liberated electrons are then available at various points in the gas due to the initial avalanche that is still advancing toward the anode. The photoelectrons will be accelerated by the electric field and can cause avalanches of their own. These new avalanches can repeat the whole process and thus account for the short formative times involved in the streamer breakdown process [Ref. 6].

The positive ions, due in part to their much larger mass, will be virtually left behind as the electron avalanche advances. They constitute the highly ionized channel that is characteristic of the spark. Those ions act to carry further electrons which sustain the discharge.

This streamer mechanism of breakdown is much faster than that of the Townsend mechanism. Single avalanches can produce streamers which proceed at a much shorter time than the electron crossing time. This accounts for the short formative time lags [Ref. 7].

Streamers are the forerunners of the breakdown process. In a uniform electric field, streamer formation always leads to breakdown. In

fact, breakdown can also occur without the development of streamers. It has been shown [Ref. 8] that there is a minimum avalanche size, below which streamers do not form.

As already stated, there are only two known methods which are responsible for breakdown. Experiments show [Ref. 9]. that there is no abrupt change that would signal a transition from one type to the other. Often the discharge follows the Townsend pattern followed by a streamer occuring near the anode. This streamer then propagates to the cathode causing breakdown. This pattern is best explained by considering the cloud of positive ions which is left behind the advancing avalanche. This distorts the local electric field in such a manner as to enhance the avalanche effect.

The basic difference between breakdown in a nonuniform field in contrast with a uniform field is the fact that streamers which develop in a uniform field always lead to breakdown. In the nonuniform case, however, a variety of events take place before the breakdown occurs. The first experimenters were interested in the glow that would appear before the breakdown. These glows, or better yet glow-discharges, were either self-sustaining or transient depending primarily on the degree of electric field nonuniformity. These coronas have been observed naturally during electrical storms and as "St. Elmo's Fire". They generate a considerable amount of electrical noise as well.

The transition to glow from intermittent streamers is thought to be due to the development of a space charge around the anode. This space charge creates somewhat of a uniform electric field in this region.

This can lead to a breakdown within this gap which is responsible for the glow. This type of discharge is relatively stable due to a variety of mechanisms [Ref. 10]. Such glow will only develop in the presence of negative ion formation. Corona glow raises the arcing threshold voltage which would otherwise lead to a streamer-to-arc breakdown transition.

When voltage is applied to electrodes so as to create a highly nonuniform electric field, the first observable phenomenon is that of a branched filament already identified as a streamer. The more uniform the electric field, the less branching of the anode streamer. Experiments shown that the branches never cross and tend to repel each other [Ref. 11]. The number of branches increases from the cathode to the anode. This increase is exponential. The anode streamer channels can thicken due to new growth. The glow from these streamers comes only from the tips which are advancing. This is where the new ionization is taking place.

Electron emission from the cathode can take place when the striking anode streamers are of sufficient intensity. The emission is due to the large electric field of the space charge in front of the anode streamer tips. Electrons accelerated toward the streamer can create avalanches of new electrons by collisions with the neutral gas. Thus the conductivity is rapidly enhanced and a jump in the current between the electrodes is observed.

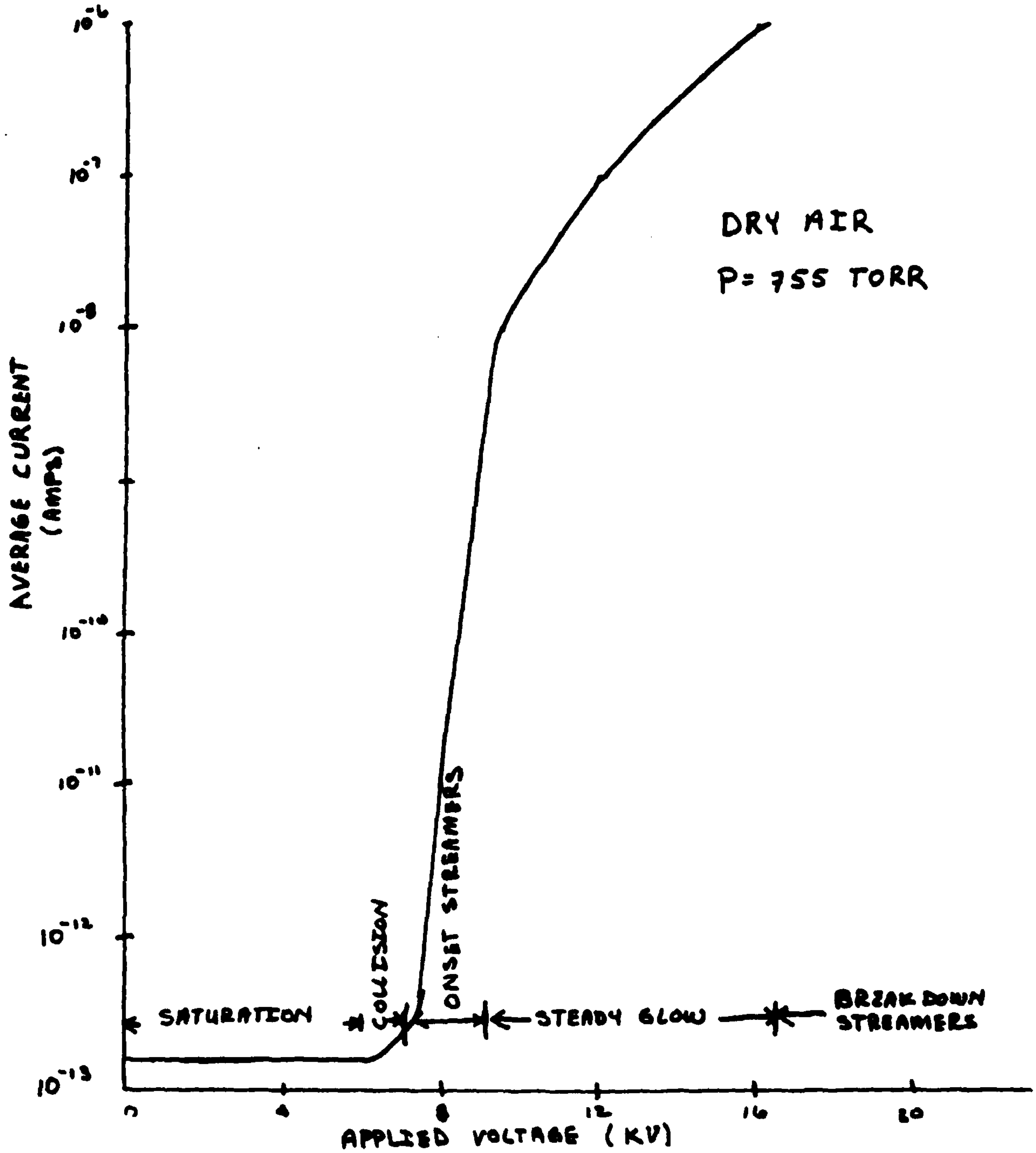
A fully ionized plasma behaves differently than a partially ionized in many ways. One principle difference is in the conductivity. It can be shown [Ref. 12] that the conductivity of a fully ionized plasma is

proportional to the electron energy to the 3/2 power. However, in a partially ionized plasma, the conductivity is dependent on the charged particle density directly.

The conductivity of a streamer is not very high. Therefore, breakdown may not occur when the streamer channel reaches the cathode. It is the electrons emitted from the cathode, propagating through the channel, that increases the overall conductivity.

When the voltage is slowly applied to the electrodes, a few streamer bursts occur. These are principally due to random fluctuations in the gas. As the voltage is increased further, streamers develop more often and eventually lose thèir fluctuating nature. When the transients die away, a steady glow appears near the anode surface. The current between the electrodes is continuous with small fluctuations superimposed on it. With increasing voltage and hence increasing current, the glow increases in both intensity and size [Ref. 13]. Just prior to the onset of breakdown, a streamer type transient discharge appears with the associated increase in anode glow brightness and electrical noise. This is called a breakdown streamer whereas the former is known as an onset streamer.

Figure 1 [Ref. 14] shows the different current-voltage regions which occur in dry air at atmospheric pressure. The electric field is highly non-uniform due to special electrode arrangements. The first region is the current produced by natural ionization of the gas due primarily to cosmic rays. The next region is that caused by ionization due to collisions. This is followed by increased current from the onset streamers. When the sharp current increase slacks off, a steady glow



Typical I-V Relationship for an Anode Corona at a Point-to-Plane Electrode Arrangement. The Point Radius is 0.5mm and the Gap Spacing is 8 cm.

Figure 1. Dry Air Voltage-Current Diagram [Ref. 14]

forms which transitions into breakdown streamers and finally a spark or breakdown with increased voltage.

Breakdown streamers have been seen to have a slow rise and longer decay time associated with them when compared to onset streamers. These pulses occur with a frequency from zero to three to four thousand pulses per second. Beyond this the frequency decays but the streamer duration increases.

Near a cathode that has a highly nonuniform electric field, similar events occur which lead to cathode glow [Ref. 15].

III. AMBIPOLAR DIFFUSION

Due to a density gradient within the plasma channel, there will be an exodous of mass due to diffusion. If the electron density is low, as in a partially ionized gas, each species of particles may be considered independently. However, this may not be the case in the highly ionized spark channel.

Diffusion prevents a well defined boundary between the highly ionized channel and the slightly ionized gas which surrounds it from existing for all but very short times. If the distribution of electrons and ions is not well behaved, (as in the presence of magnetic fields), then the electrostatic repulsion among like particles will assist in the diffusion process. This is precisely the case that occurs when there is a build-up of either electronic or ionic space charges. The accurate treatment of electrostatic repulsion is beyond the scope of this work and will not be treated further.

To create a steady-state discharge situation, the need exists to not only ionize the gas, but also to de-ionize the gas. The rate of ionization must equal the rate of de-ionization. The loss of electrons from the spark channel occurs principally in two ways: diffusion out of the system or recombination of an ion with an electron. These lead to the direct de-ionization of the plasma.

Electron attachment to a neutral atom may also occur, especially in a medium with electro-negative atoms such as oxygen. The negative charge is not lost to the system; however, the large increase in the

mass of the body reduces the charge's mobility by several orders of magnitude. Positive ions may perform the same process which is called cluster formation. Since the mobility of the ion is so much smaller than the electron to begin with, the overall mobility of the cluster formation is not reduced to the degree that the attachment process slows the electron.

Collisions between like particles give rise to no diffusion. This situation is to be contrasted with the case of ions colliding with neutral atoms. The final velocity of the neutral is of no concern and the ion random-walks away (diffuses) from its initial position. Ion-ion collisions have a balance in each collision. That is, for each ion that moves out, the other moves in as a result of the collision.

When an electron and an ion collide, however, the situation is different. Unlike particle collisions give rise to diffusion. The physical picture is difficult due to the disparity in mass between the electron and ion. The ions remain relatively stationary and the electrons do the random walk [Ref. 16].

Particles that are able to diffuse to the wall of the container are able to recombine and are thus lost to the system. Therefore, within the gas the electrons can thermally diffuse to the walls, recombine within the plasma, create negative ions, or undergo ambipolar diffusion.

Under the influence of thermal forces, the electrons' smaller mass than the ions' means thay can diffuse faster than the ions. In the process, the electrons leave behind an excess number of ions which creates a local space charge and thus an electric field. This microscopic field will then tend to retard the motion of further electrons

and propel the ions. The two types of charge carriers will diffuse as one. This is called ambipolar diffusion [Ref. 17]. The effect of the ambipolar electric field is to enhance the diffusion of ions by a factor of two, but the diffusion rate of the electrons and ions together is primarily controlled by the slower species.

Electrons which cannot propagate to the electrodes will be lost through recombination, diffusion or attachment. The charge carriers will diffuse throughout the medium if there exists a density gradient in much the same manner as neutral atoms if there are no forces to confine them. When they reach the wall, they recombine and are lost to the gas.

The current density due to diffusion is [Ref. 18].

$$J = -eD\,\nabla n \tag{1}$$

The rate of change of particle density due to diffusion is:

$$e\left.\frac{\partial n}{\partial t}\right)_D = -e\,\nabla\cdot[D\,\nabla n] \tag{2}$$

If we assume that all ionization mechanisms have stopped, then the rate of change of electron density due to diffusion is:

$$\frac{\partial n}{\partial t} = \nabla \cdot [D \nabla n] \qquad \text{[Ref. 19]} \qquad (3)$$

In many gaseous discharges the charged particles are contained within a cylindrical channel. This is the exemplified in the fluorescent light tubes. If the channel is much longer than it is wide, then the end effects may be neglected. In addition, the radius of the cylinder must be larger than the Debye length. The analysis of the effects of diffusion and recombination are therefore done in a cylindrical geometry. Moreover, the diffusion coefficient (D) is simply treated as a constant in equation (3). Note that the net current in ambipolar diffusion is zero as the electron and ion diffuse together. However, the ambipolar particle density change is driven by density gradients rather than electro-static forces.

IV. RECOMBINATION

The term recombination refers to a charge neutralizing encounter between charge carriers of opposite sign moving about in a gas [Ref. 20]. The first studies of this effect were made by Thompson and Rutherford in 1896. They stated the principles of recombination to explain the reduction of conductivity in a partially ionized gas.

The recombination coefficient is a measure of the number of recombining events per unit time and volume. It may be related to a recombination cross section [Ref. 21].

When an electron and an ion interact or collide at relatively low velocity, they have a finite probability of recombining into a neutral atom. In order to conserve momentum, a third body must be present. This body may be another particle or photon. If it is a photon, then it is called radiative or two-body recombination. When another particle is involved, it is called three-body recombination. This represents a loss term to the continuity equation [Ref. 22]. Two-body recombination predominates when the gas pressure is below a few Torr.

Due to losses of electrons through diffusion, attachment and etc., an accurate value of the recombination coefficient is difficult to obtain experimentally. Biondi [Refs. 23 and 38] has found that the time and pressure dependencies of electron density in certain gases follow a two-body recombination law over a wide range of variables. The gas must be quite pure, however, to reproduce his results as the formation of

negative ions and the Penning effect can completely destroy any sense in the experimental data.

Complications arise when the effects of diffusion are included in the recombination measurements. Some of these effects have been analyzed for infinitely long cylinders [Ref. 24]. One important conclusion is that linearity of the plot of one over the electron number density versus time does not imply a meaningful value for the recombination coefficient derived from the slope of the plot. It turns out that linearity may result even when diffusion is controlling the decay process. Either the diffusion loss term or the recombination loss term may be dominant over most of the plasma at any particular time (except at late times in the afterglow when diffusion must dominate the electron loss) [Ref. 25]. Therefore, the plasma may be either diffusion or recombination controlled at any given time. If one of the two mechanisms dominate for a sufficiently long period of time during the decay, then the electron distribution will follow a solution which would occur if the other mechanism were absent. (See discussion in Chapters V and VI on linear and non-linear solutions of the electron decay equation.)

J. J. Thompson found experimentally that diffusion to the walls alone could not account for the reduction of conductivity of an ionized gas. The recombination of a free electron with an ion was a significant factor in the neutralization process.

Recombination between an electron and an ion can take place only when the electron is in the vicinity of the ion for about 10^{-8} seconds. Clearly, only slow moving electrons will be able to recombine with the

ion as it needs to remain in the vicinity of the ion. During two body recombination, the electron loses excess energy through photon emission. Otherwise, three body recombination where the electron transfers excess energy to another electron is the dominant process. The collision cross section for the electron-electron collision depends on the electron temperature and is thus an unlikely event in normal gases. When looking at collision times, if (τ_{en}) is the smallest, then the neutral particles determine the recombination rate of the gas. This is determined purely by species pressure. If (τ_{ei}) is smallest, then forces other than species pressure determine recombination rates. For a neutral interaction, the collision must occur within the Bohr radius. However, for charged particles, the Debye radius will be the determining factor. Since the Debye radius is much greater than the Bohr radius, the charged particle interaction will dominate.

When ionizing a neutral gas, the density of electrons and ions should be the same for single ionization. The number of recombinations per second is then:

$$\left.\frac{\partial n}{\partial t}\right)_R = -\alpha\,[n_e \cdot n_i] \tag{4}$$

$$= -\alpha n^2 \tag{4a}$$

where alfa is the constant of proportionality. Alpha is in fact not a constant [Ref. 26]. Consider the case where the ionizing source is turned off. Many of the free electrons will still be near their former atom. The probability of electrons recombining with the ions is large. Therefore, the recombination coefficient will initially be large and then become smaller with time.

It at time zero, the density of charge carriers is n_0, one can integrate the above equation directly. This becomes:

$$\int_{n_0}^{n} \frac{\partial n}{n^2} = \int_0^t -\alpha \, \partial t \tag{5}$$

which has the solution:

$$n = \frac{n_0}{1+\alpha n_0 t} \tag{6}$$

This is called the ion loss formula [Ref. 27].

Recent investigations show that the number of charge carriers produced by ionization may be different due to attachment, multiple ionization and etc. Also it has been found that two different ions can exist at the same time and may have different recombination rates [Ref. 28].

V. LINEARIZED BREAKDOWN EQUATION AND SOLUTION

A model of the streamer-breakdown event is necessary to adequately describe the physical events which occur. The streamer is thought to be a channel or tube of high charge concentration which leads to high conductivity. This channel of ions supports the flow of electrons from the cathode to the anode during the arc transition. The effects which reduce the ion concentration and the electron density have already been described as being two body recombination, diffusion, convection and electron attachment. Many more factors need to be included in any complete description of the events that take place within the streamer tube (such as three-body recombination).

Radiative recombination (two-body recombination) requires a photon to carry away excess energy from an electron-ion recombination. In three-body recombination, a third body removes excess energy. This third body may either be an electron or some heavy particle. An electron may also recombine with a molecular ion. If the excess energy goes into the dissociation of the molecule, this is called dissociative recombination. Another way of removing excess energy is to form a neutral atom in which two electrons are simultaneously excited. This is known as dielectronic recombination.

The total time rate of change of the charged particle density may thus be written:

$$\frac{\partial n}{\partial t} = \frac{D_a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) - U\frac{\partial n}{\partial r} - \alpha_2 n^2 - \alpha_3 n^3 + \ldots \tag{7}$$

where the first term on the right is the diffusion term, followed by the convective term, the two body recombination and three body recombination terms. (D_a), (α_2) and (α_3) are the respective coefficients. U is the flow velocity of the gas. The geometry is shown in Figure 2.

First, an order of magnitude test is made to determine the relative importance of these terms. To do this, characteristic times are defined and their magnitudes compared. Nominal values were used from Table 1 for the above coefficients for nitrogen at atmospheric pressures.

Actual values of the above coefficients are dependent on the ionic species in the gas. It is still possible to obtain general order of magnitudes for the coefficients, however. The values shown were taken from Mitchner and Kruger [Ref. 30]. Because ($\tau_{CON} >> \tau_r$) the convective term is neglected.

These characteristic times show that the effect of recombination is considerably faster than all but turbulent flow on the alteration of charged particle density. For a first order approximation, then, one could assume negligible convective and diffusion effects if the fluid flow were laminar. This has already been done without stating so in the chapter on recombination. There the solution was found to be:

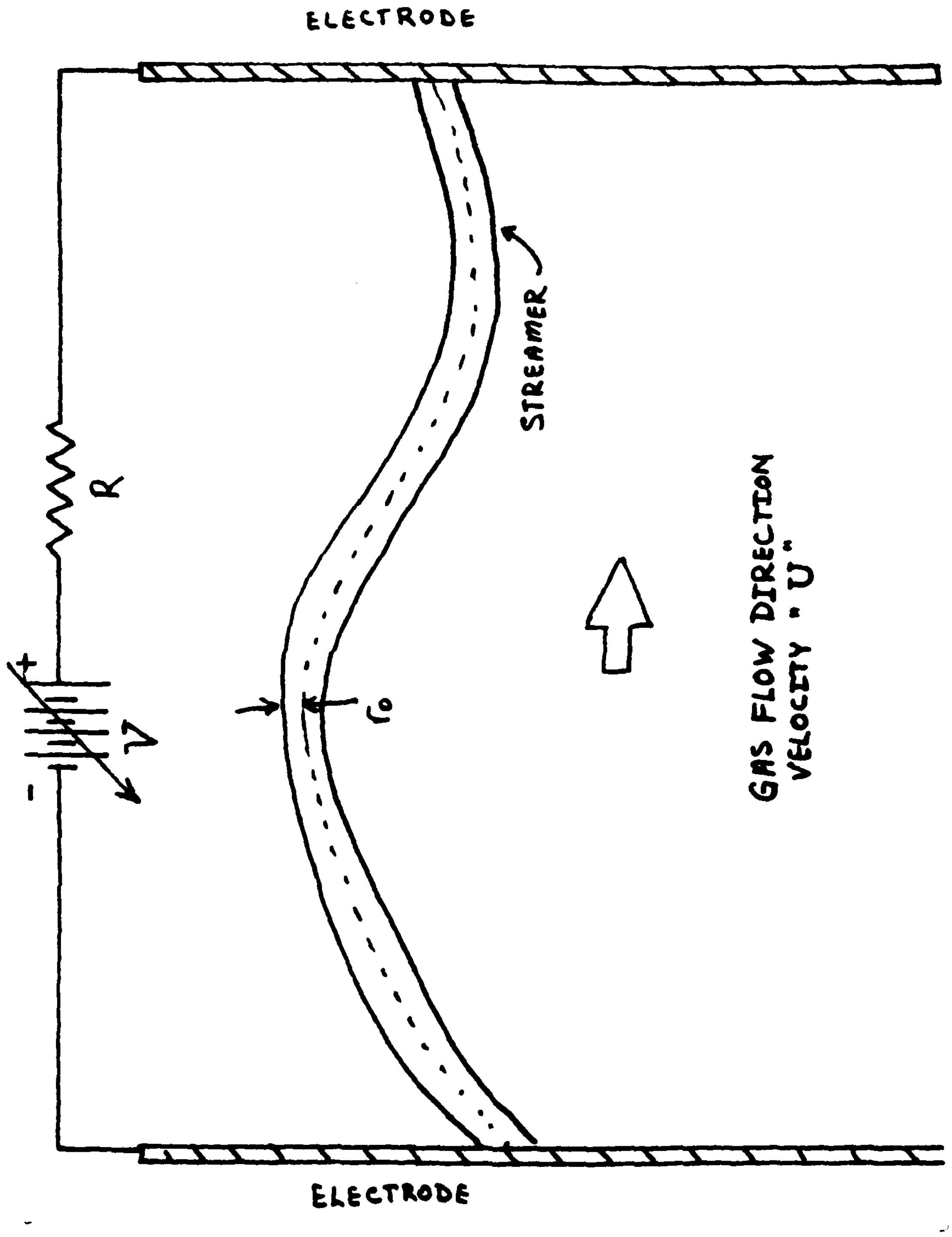


Figure 2. Streamer Geometry

Table 1. Coefficient Values for Nitrogen [Ref. 29]

N_o	$= 10^{20}$	m^{-3}
α_2	$= 10^{-13}$	m^{3}/sec
α_3	$- 10^{-35}$	m^{3}/sec
r_o	$= 10^{-4}$	m
D_a	$= 10^{-4}$	m^{2}/sec for laminar flow
D_a	$= 10^{-1}$	m^{2}/sec for turbulent flow
L	$= 10^{-1}$	m
u	$= 10^{2}$	m/sec
τ_{conv}	$= L/U$	$= 10^{-3}$ sec
τ_{diff}	$= R_o^{2}/D_a$	$= 10^{-4}$ sec for laminar flow
τ_{diff}	$= R_o^{2}/D_a$	$= 10^{-7}$ sec for turbulent flow
τ_{recom}	$- 1/\alpha N_o$	$= 10^{-7}$ sec

$$n(t) = \frac{n_0}{1+\alpha n_0 t} \tag{8}$$

with the boundary conitions of:

$$n(0) = n_0 \quad \text{and} \quad n(\infty) = 0 \tag{9}$$

If the flow is fully turbulent, the diffusion term may not be neglected in the calculations. Therefore, let us assume that the gas flow is in some intermediate state; neither laminar nor turbulent. Turbulent fluid flow may increase the diffusion coefficient by three or more orders of magnitude. The characteristic diffusion time will then be on the same order of magnitude as the recombination time.

The equation which models the situation for two-body recombination is:

$$\frac{\partial n}{\partial t} - \frac{D_a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = -\alpha n^2 \tag{10}$$

The three-body analysis is given in Appendix A. To achieve a linear approximation of this equation we must insure the flow is only mildly

turbulent. With this in mind, diffusion may be introduced as a "perturbation" of the recombination equation. Equation (10) can then be linearized because if the perturbation is small, then the non-linear terms can be neglected. First the charge density is written in two parts: a base and a perturbation part. The base charge density expresses the state of the plasma channel in the absence of the diffusion. The linear theory is valid as long as the diffusion term is much smaller than the recombination term. This is true for non-turbulent situations. To include the fully turbulent case, a numerical solution of the governing equation must be done. This will follow in Chapter VI.

The charge carrier density is now written in the form:

$$n(r,t) = \tilde{n}(t) + \varepsilon n_1(r,t) \quad \text{where } \varepsilon \ll 1 \tag{11}$$

In order to get an analytic solution and make the derivation easier to follow, we shall switch from the axisymmetric or polar case to a two dimensional solution. The governing equation is now:

$$\frac{\partial n}{\partial t} - D_a \frac{\partial^2 n}{\partial x^2} = -\alpha n^2 \tag{12}$$

where now we let:

$$n(x,t) = \tilde{n}(t) + \varepsilon n_1(x,t) \tag{13}$$

This is substituted into the governing equation:

$$\frac{\partial \tilde{n}}{\partial t} + \varepsilon \frac{\partial n_1}{\partial t} - \varepsilon D_a \frac{\partial^2 n_1}{\partial x^2} = -\alpha\left[\tilde{n} + \varepsilon n_1\right] \tag{14}$$

$$= -\alpha\left[\tilde{n}^2 + 2\varepsilon \tilde{n} n_1 + \overset{\text{H.O.T.}}{\cancel{\varepsilon^2 n_1^2}}\right] \tag{15}$$

Now we may neglect the higher order terms and also cancel the base part:

$$\frac{\partial n_1}{\partial t} - D_a \frac{\partial^2 n_1}{\partial x^2} = -2\alpha \tilde{n} n_1 \tag{16}$$

$$= \frac{-2\alpha n_1 n_0}{1 + \alpha n_0 t} \tag{17}$$

The linearized solution to equation (10) is:

$$n(x,t) = \frac{n_0(1-\varepsilon)}{1+\alpha n_0 t} + \frac{\varepsilon n_0}{2(1+\alpha n_0 t)^2}\left[\operatorname{erf}\left(\frac{x_0+x}{2\sqrt{D_0 t}}\right) + \operatorname{erf}\left(\frac{x_0-x}{2\sqrt{D_0 t}}\right)\right] \tag{18}$$

$$\text{for} \quad x \leq x_0$$

$$= \frac{\varepsilon n_0}{2(1+\alpha n_0 t)^2}\left[\operatorname{erf}\left(\frac{x_0+x}{2\sqrt{D_0 t}}\right) + \operatorname{erf}\left(\frac{x_0-x}{2\sqrt{D_0 t}}\right)\right] \tag{18a}$$

$$\text{for} \quad x > x_0$$

The error function correctly models the step decrease in the electron density at the boundary at time zero. The utility of the error function is also demonstrated at times greater than zero. The function allows the effusion of electrons across the boundary of the ionized channel. This process models the effect of diffusion while still meeting the requisite boundary conditions. A plot of the error function in non-dimensional parameters is shown in Figure 3. The three-body counterpart of equation (18) is given in Appendix A.

The diffusion coefficient appears only in the argument of the error function. The effect of larger diffusion coefficients is to increase the rate at which the electrons leave or diffuse from the original plasma channel. If the diffusion coefficient were zero, then the electrons would never cross the initial boundary of the plasma channel. That is, the form of the channel would be independent of time. However,

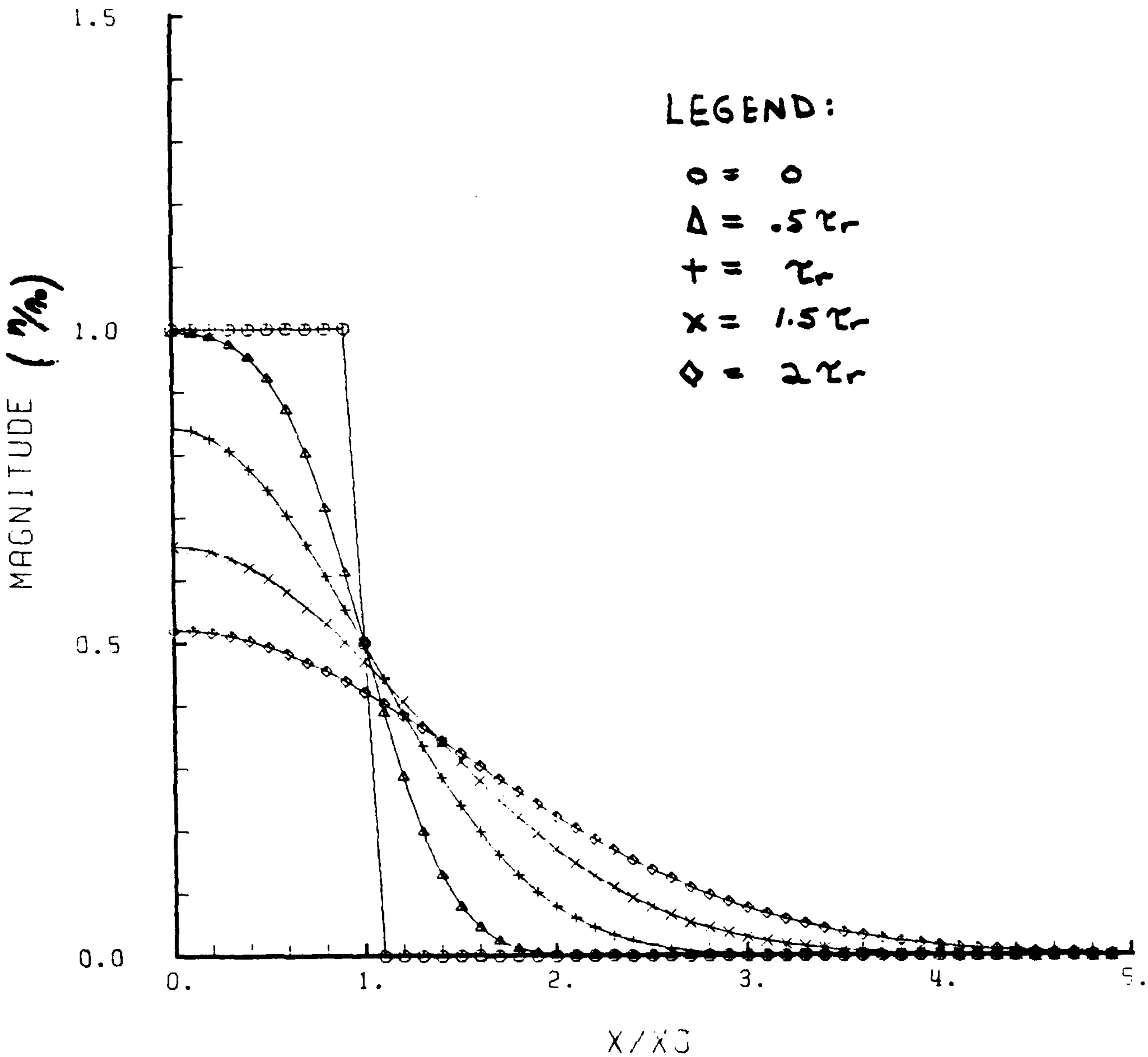


Figure 3. Plot of Error Function (ERF)

the electron density within the channel would decrease in time due to the recombination term. These numerical results are in keeping with the physical picture of the plasma channel.

It must be remembered that all discussion pertaining to the diffusion of electrons really means the ambipolar diffusion of the electrons and ions together. This was described in Chapter III.

VI. THE NON-LINEAR EQUATION AND SOLUTION

Non-linear equations are defined as those which contain products of the dependent variables or transcendental functions. One can solve these equations either for the "roots" or for solutions which satisfy a set of non-linear equations. The solution to equation (7) which models the de-ionization of the plasma channel seeks the roots.

There appears to be no solution in terms of known functions to the general form of equation (7). Sometimes it is not clear whether a given equation has a solution at all. The solution of the non-linear form is of an interative nature. To ensure convergence, it was necessary that a reasonable approximation to the solution in question be known prior to iterating [Ref. 31]. That was the purpose of the linearized solution presented in the preceeding chapter. If the solutions at all match, then there may be some degree of confidence that the solutions are correctly describing the situation. If there is a great disparity between the two solutions, then the reason for the discrepancy must be resolved.

Because the geometry of a streamer is cylindrical in nature, the cylindrical coordinate system is used in the numerical analysis. The system is then described by two dimensions; radial position and time. It is assumed that there exists angular symmetry and that the length of the ionized channel is much greater than it's diameter. This permits the exclusion of electrode effects at either end.

The specification of the boundary conditions was not without some ambiguity. Boundary conditions are specified along the centreline of the channel and at infinity with a finite-difference computational method.

In the computational model, the two-dimensional axes represent time in the vertical and radial position along the horizontal. At time zero, the electron density in the radial direction is presumed to be constant out to some point r_o. After this point, the electron density is zero.

Along the centreline of the plasma channel, the slope of the electron density versus position must be zero. This is used to define the starting value for the electron density along the centreline for any time greater than zero.

Knowing these boundary conditions, the geometry of the situation lends itself well to a finite difference computational molecule. The grid size in the radial direction is relatively insensitive in the solution of the equation. A convenient step size of $r_o/10$ is used. The same is not true for the step size in the time dimension. The primary reason for this leads back to the characteristic times encountered in the linearized solution. The timewise step size needs to be smaller than the smallest characteristic time of the system. In this case it turned out to be the recombination time. The timewise step size was thus taken to be $\tau_{rec}/10$.

The technique used along with the computational molecule is the explicit method of temporal development [Ref. 32]. The principal defect in this form of solution is the need for very small time increments to

ensure stability. This leads to lengthy computation. A reasonable modification to this solution to extend it to greater times without the need for many iterations would be to go to an implicit recurrence formulation. This is where two or more unknown values of the equation at t(i+1) are expressed in terms of known values. This can be done by the solution of sets of simultaneous equations, which complicate the solution considerably.

What was originally deemed a "stability border" was found in both the linear and non-linear solutions. This border governed the size of the time steps. If the delta time was too large, solutions which contained physically meaningless densities (negative values) were the result. This border was found to be roughly one-fifth to one-tenth the shortest characteristic time in the system. This empirical finding lead to the development of the one-tenth of the recombination time as the time step interval. While researching the literature, this small time step was shown to be a necessary consequence of the numerical method employed as previously stated.

If one considers a cartesian coordinate system where the 'X' axis represents radial position and the 'Y' axis is time, the values for the electron density can be established using a "marching" process [Ref. 33]. The boundary conditions specify the values of "n" (electron density) along the radial position at time zero. The requirement of zero slope of the density curve at the centre of the plasma 'tube' for any time is a second requirement. Thus, n(r,0) is specified. The density at other points is then determined by the density at preceeding points.

With the marching scheme in mind, the numerical evaluation of the points on the graph are done in the following manner:

i = radial position marker

j = time increment marker

$$\left.\frac{\partial n}{\partial t}\right)_{i,j} = \frac{1}{\Delta t}\left\{ n_{i,j+1}, n_{i,j} \right\} \tag{19}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = \frac{1}{(\Delta r)^2}\left\{ n_{i-1,j} - 2n_{i,j} + n_{i+1,j} \right\} + \frac{1}{2(\Delta r) r_i}\left\{ n_{i+1,j} - n_{i-1,j} \right\} \tag{20}$$

There is a restriction that is easily met in this solution where:

$$\Delta r^2 > 2 D_a \Delta t \qquad \text{[Ref. 31]} \tag{21}$$

Because of the small time steps used, this restriction has no effect on the calculations unless the user reduces the radial step size to very small steps. In the physical picture, the initial radius of the dis-

charge is on the order of a few Debye lengths. However, to reduce the step size much smaller than one-tenth to one-hundredth of the discharge radius, the time interval will have to be adjusted accordingly.

These two parts of the non-linear equation are the only ones that need modification to perform the numerical solution. The recombination terms need no adjustments as there is no time or positional derivatives associated with them. They depend principally on the charge density of the system.

The governing equation which is originally written as:

$$\frac{\partial n}{\partial t} - \frac{D_a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = -\alpha n^2 \tag{22}$$

may now be written:

$$\frac{1}{\Delta t}\left\{n_{i+1,j} - n_{i-1,j}\right\} - \frac{D_a}{(\Delta r)^2}\left\{n_{i+1,j} - 2n_{i,j} + n_{i-1,j}\right\} - \frac{D_a}{2r(\Delta r)}\left\{n_{i+1,j} - n_{i-1,j}\right\} = -\alpha n^2 \tag{23}$$

This is the form used in the numerical solution. The program defines the initial conditions and boundary conditions before iteration begins. It then progresses along the radial position axis at a fixed time increment based upon the shortest characteristic time in the system

and evaluates the charge density at every point and stores it in an array. When the program has determined the charge density at all points in the array, it then proceeds to plot the densities in one of several outputs specified.

All constants in the program may be changed. The only care that need be observed is when adjusting any parameter which changes the characteristic recombination or diffusion times. The time-wise iteration steps are based on the recombination time as is the diffusion time. Thus, in general, no modification to the program is required when changing initial charge density, n_o, or the recombination coefficient, α.

If one wishes to make the diffusion time shorter than the recombination time, the time-wise step will need to be modified such that it is approximately one-tenth of the shortest characteristic time of the system.

VII. EFFECT OF THE DIFFUSION COEFICIENT

Experiments have shown that the diffusion coefficient changes its value between laminar and turbulent flow. The difference is an increase of approximately three orders of magnitude. This can reduce the characteristic diffusion times under turbulent flow to the same order as the recombination time. Turbulent diffusion, therefore, could not be modeled by the linearized solution. As shown in Appendix B, the diffusion times in the linearized cases were allowed to vary between ten-thousand to ten times the recombination times. To summarize those results, one can say that in all cases, the charge density diffused outward from the plasma channel at a greater rate when the diffusion coefficient increased from laminar to turbulent.

Turbulence has a strong influence on the characteristics of electrical discharges. This is mainly due to two effects. First, the ambipolar diffusion, which also determines the rate of loss of charged particles to the walls, is greatly enhanced. In diffusion dominated, self-sustaining plasma columns, the balance is thus disturbed and this causes a substantial increase in the ionization rate, local electric fields and electron temperature.

Boundary layers seperate and mix the flow, thus increasing the transport coefficients of the plasma, in particular its heat conductivity and ambipolar diffusion. The increase in ambipolar diffusion causes the electron density profile to be more uniform over the cross section [Ref. 34].

Turbulent gas flow means that at every point in the gas the velocity fluctuates. That is, the velocity changes its magnitude and direction. When the velocity is steady, there are three fluctuating velocity components whose time average vanishes over sufficiently long periods of time.

Turbulence involves statistical fluctuations and thus, no two turbulent streams are alike. There are two average properties which are used to describe turbulence. These are the turbulent intensity and the length or scale of turbulence. When the scale of turbulence is small compared to the dimensions of the system involved (here it is the streamer radius), the turbulent intensity alone suffices to characterize the flow [Ref. 35]. This is useful in hydrodynamics and aerodynamics. This simplification is not valid in the glow-discharge situation because both glow-discharge and turbulence are microscopic properties. This tends to make precise analysis difficult at best.

As the turbulence of the free stream increases, an increase in the rate of heat transfer results due to the flow of mass from the system under investigation. The effect is remarkably great as a turbulent intensity of approximately 2.5% produces an increase in the local heat flux of about 80% [Ref. 36].

The question which may be asked here is whether the charge would recombine at a sufficiently rapid rate so as to reduce the charge density before it has a chance to diffuse out of the plasma channel. This is a result of the linear solution assumptions and the evaluation of the characteristic time of diffusion and recombination. If the recombination times are in fact several orders of magnitude faster than

diffusion, then the ionization density within the plasma channel will decrease rapidly before the charge carriers can diffuse. This will mean that diffusion is really not the dominant charge loss mechanism. This turns out to not be the case at all. In both the linear and non-linear solutions, the charge carriers diffuse out of the plasma channel long before the density drops appreciably due to recombination.

It has been previously noted in both Chapters II and III that diffusion is the dominant loss mechanism in the problem posed. This has been verified experimentally for some time [Ref. 37]. However, one must focus on the assumption that at time zero, the charge density is a step function; that is, the charge density is zero outside the plasma channel. Originally, a very steep density gradient (on the order of infinity) exists. This acts as a very strong driving mechanism for diffusion. The effects of diffusion at very short time intervals, therefore, dominate over recombination.

The results in Figures 4 through 9 show that diffusion is in fact the dominant charge loss mechanism. The figures clearly demonstrate this for all but a highly laminar flow situation. The figures all plot the log(n) versus radial position. The temporal development of the charge density profile is then shown as a family of curves. The charge loss due to recombination is somewhat lost due to the logrithmic function. This was necessary to show clearly both the reduction in charge density and its diffusion out of the plasma channel.

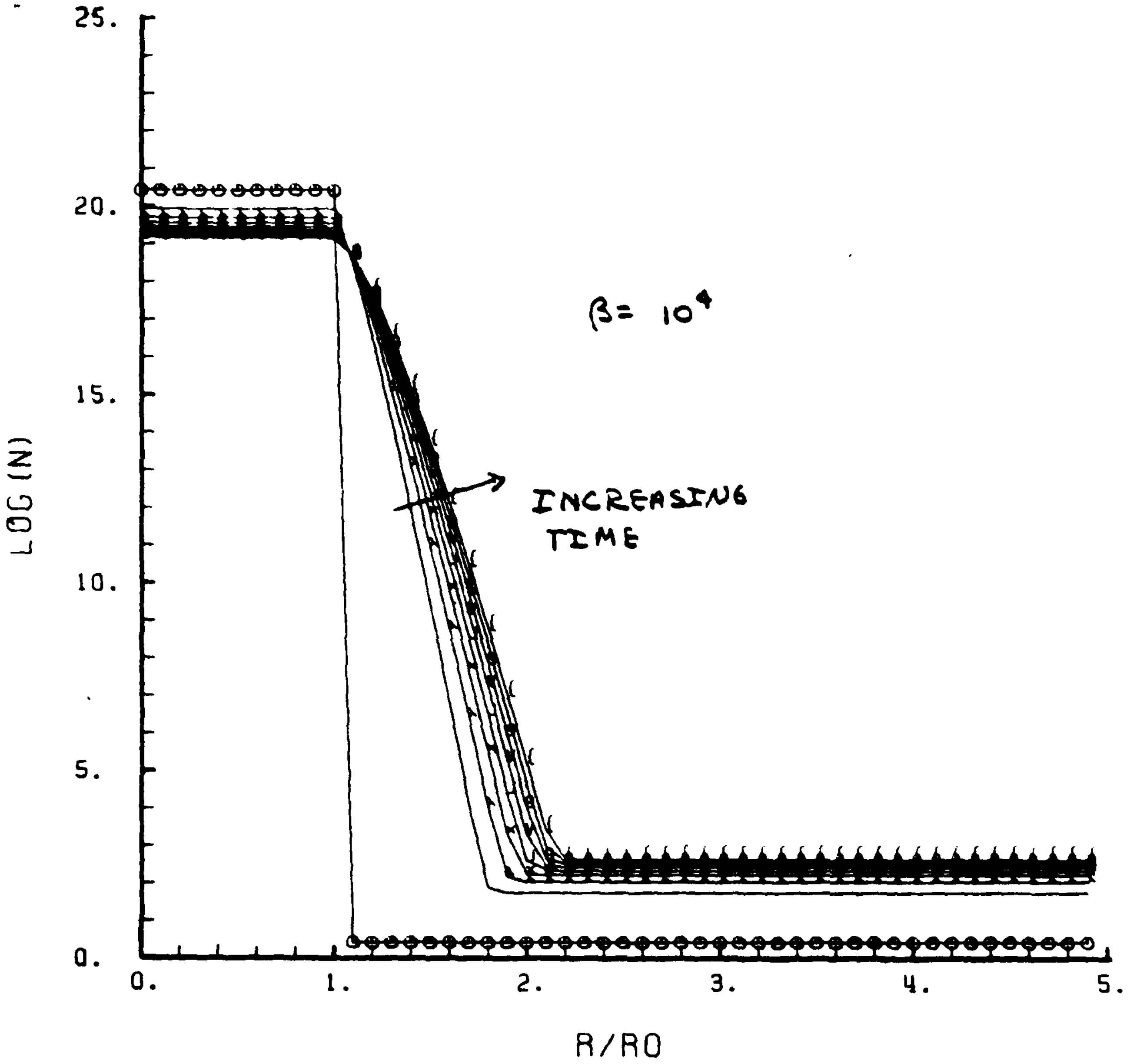


Figure 4. Results of Non-Linear Program for $\beta = 10^4$

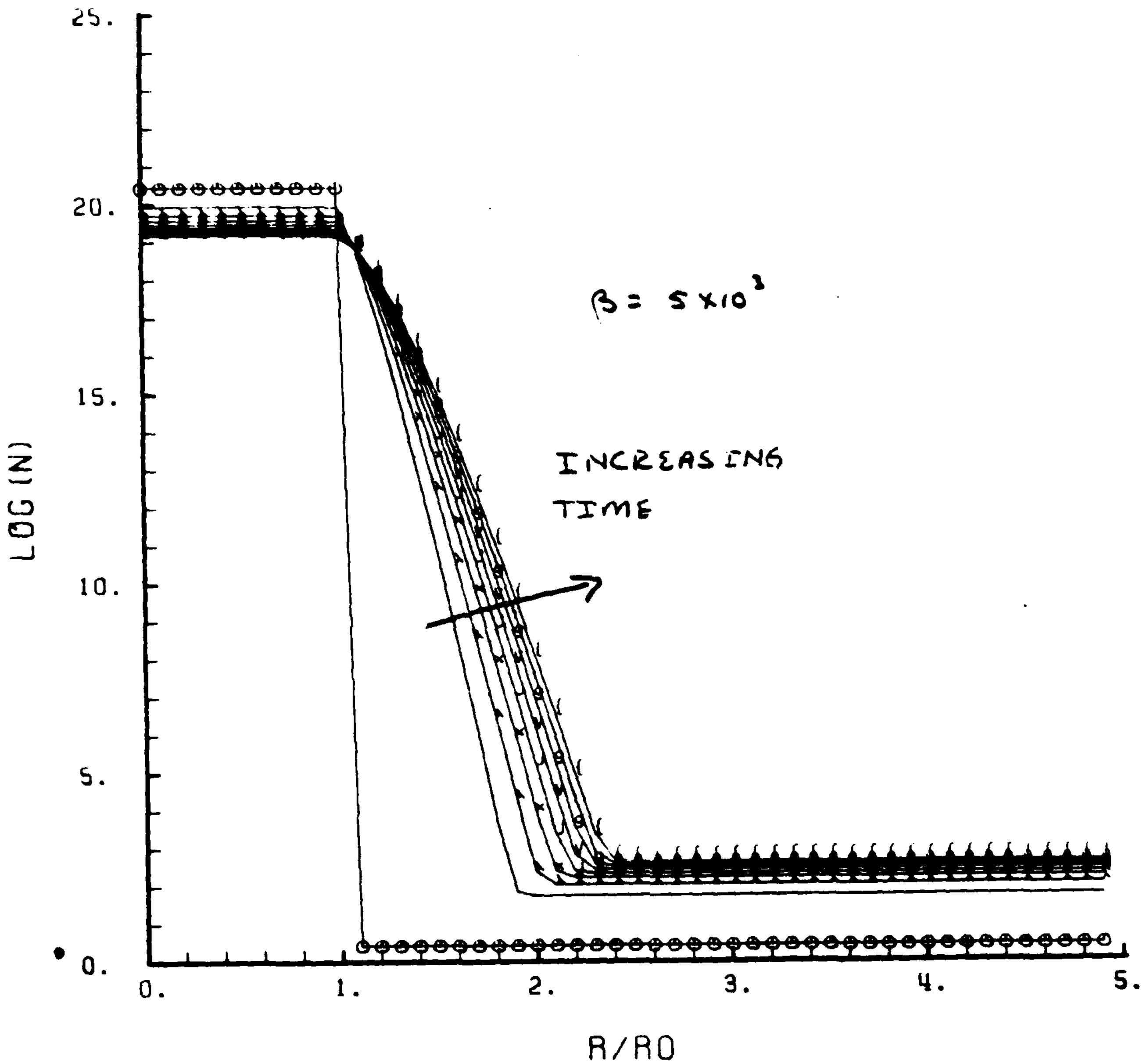


Figure 5. Results of Non-Linear Program for β = 5,000

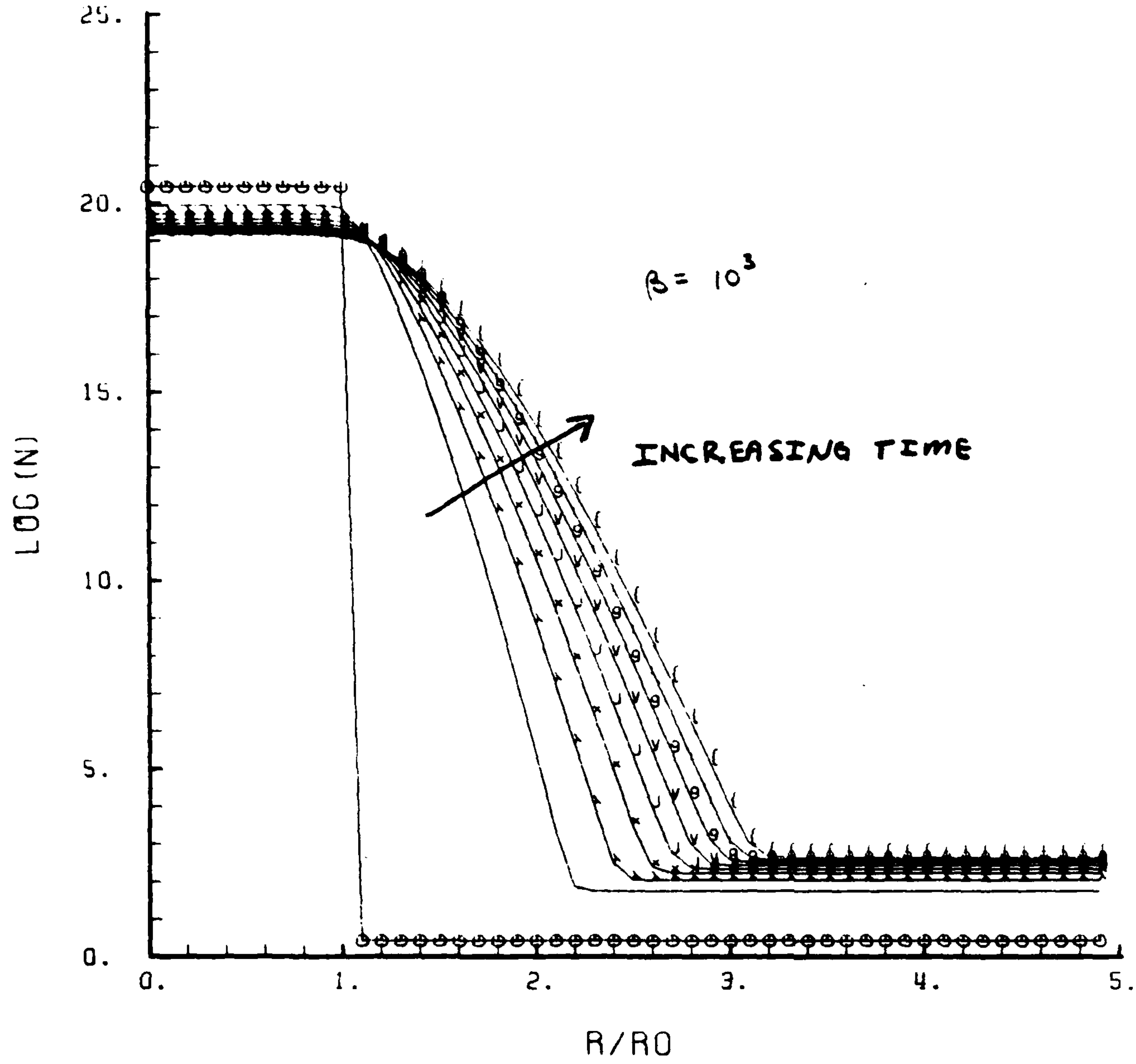


Figure 6. Results of Non-Linear Program for β = 1,000

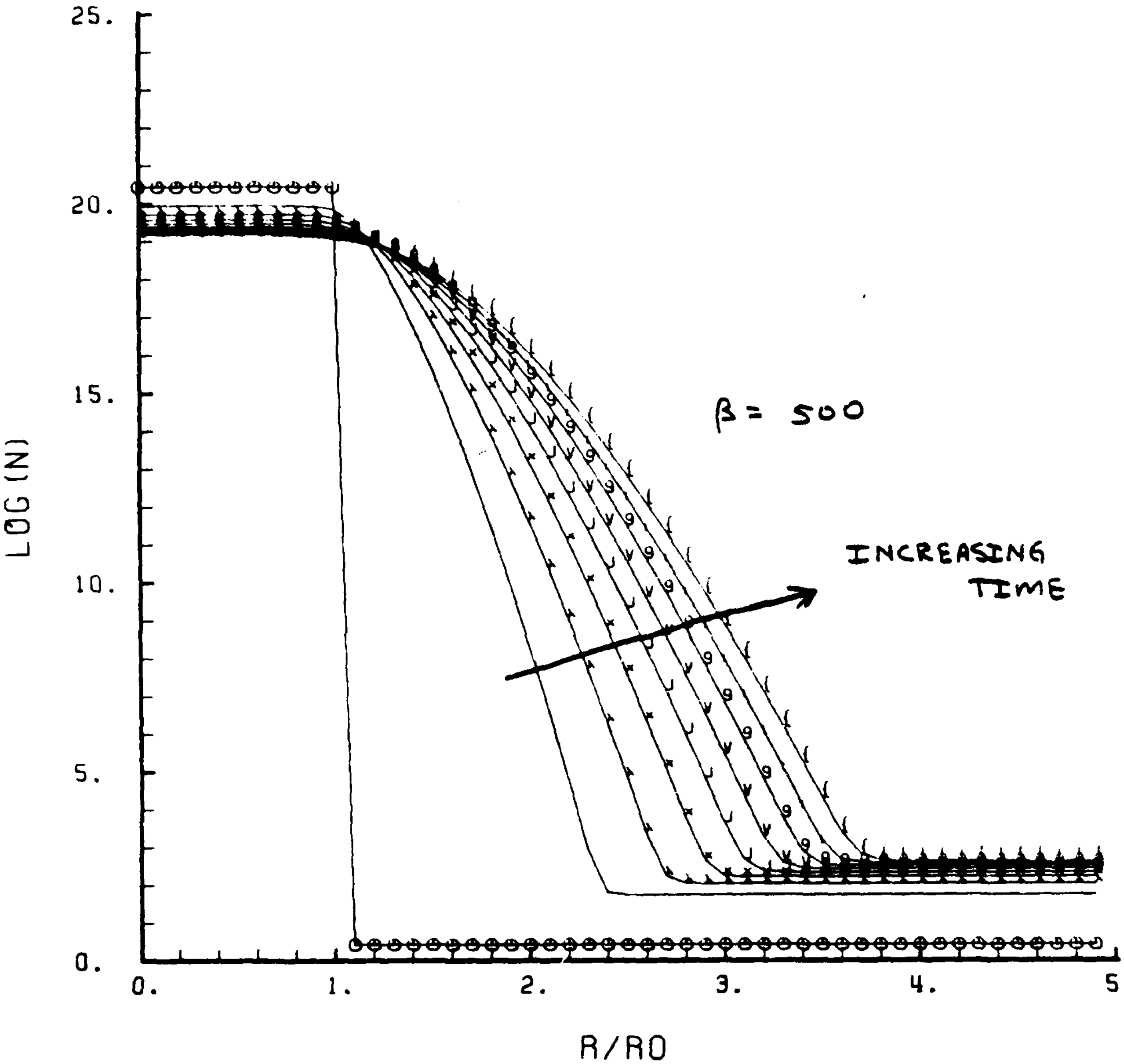


Figure 7. Results of Non-Linear Program for β = 500

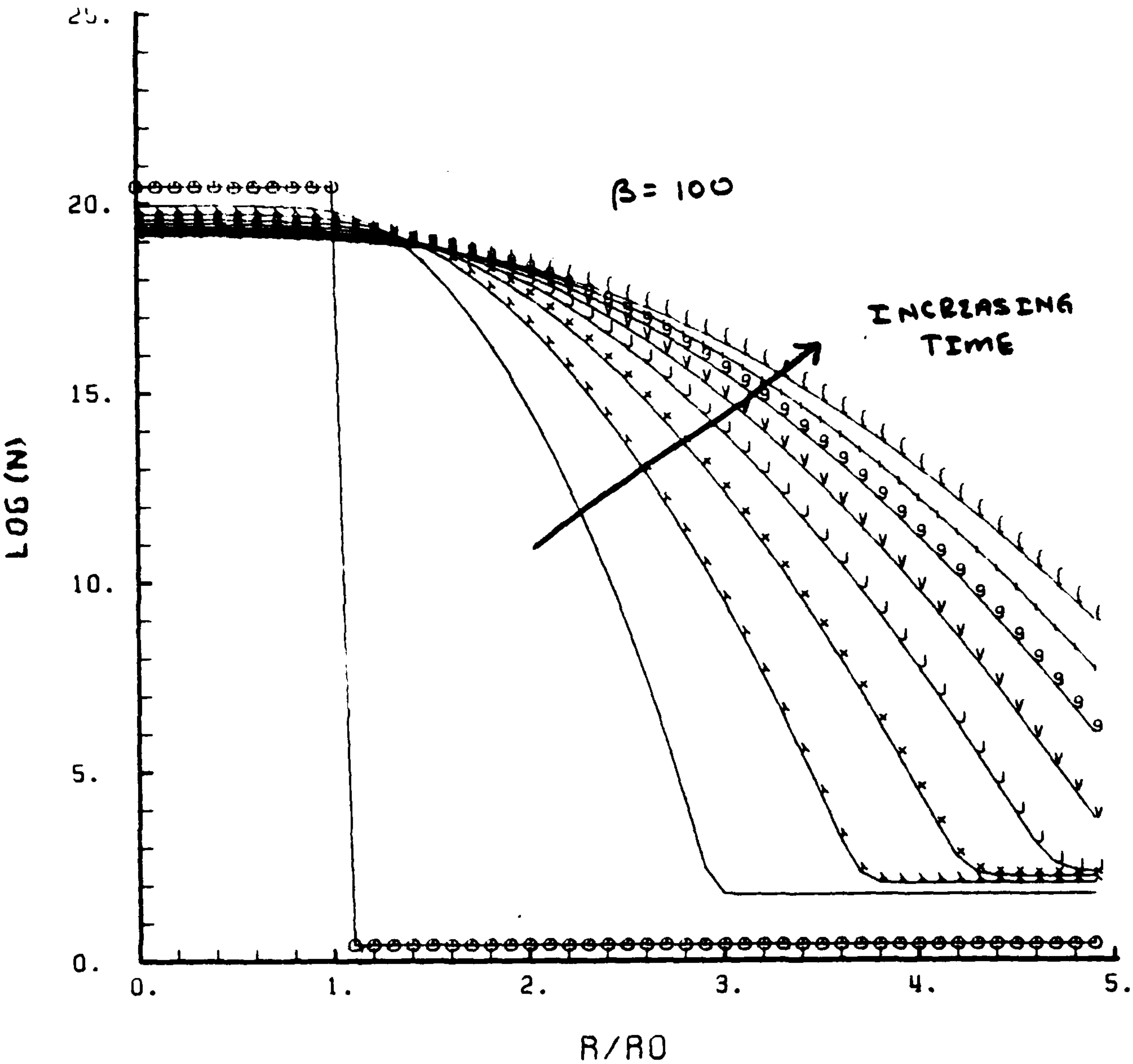


Figure 8. Results of Non-Linear Program for β = 100

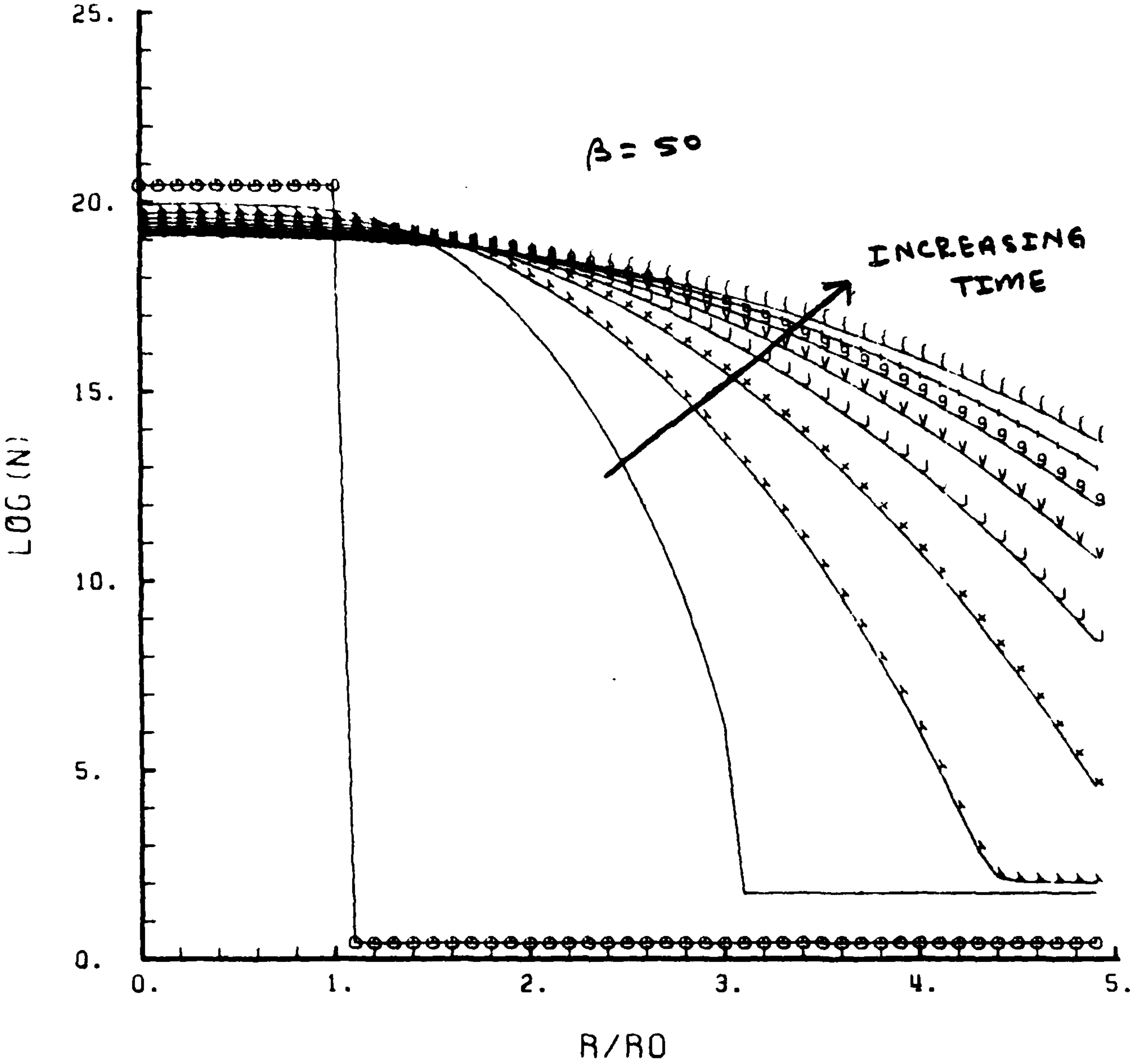


Figure 9. Results of Non-Linear Program for β = 50

Table 2 lists the plotted character and the associated time it represents on the preceeding figures. β is the ratio of τ_D/τ_R. It is characteristic of the degree of turbulence in the flow. When $\beta > 10^3$, the flow is considered to be laminar. If $\beta < 50$, then the flow is considered to be fully turbulent.

Table 2. Plotted Character Definitions

0	=	0
-	=	$2\tau_r$
λ	=	$3\tau_r$
x	=	$4\tau_r$
j	=	$5\tau_r$
v	=	$6\tau_r$
g	=	$7\tau_r$
1	=	$8\tau_r$
{	=	$9\tau_r$

The graphs on the preceeding pages are the results of the full non-linear equations as analyzed in the computer. The technique has been outlined in Chapter VI and the program used is shown in Appendix B.

All of the plots show the temporal development of the plasma cross-section. This is achieved by graphing the charge density versus radial position. The family of curves on each plot is then the charge density profile at times greater than zero. Figure 4 is for a laminar diffusion coefficient, Figure 9 is for a turbulent diffusion coefficient, and Figure 5 through Figure 8 are for arbritary intermediate levels of diffusion.

Figure 4 shows the temporal development of the charge density for a laminar gas flow. Note the very steep density gradient at time zero. This gradient acts to drive the diffusion process rapidly at first. However, as the gradient is reduced in time, the driving force is also reduced. Because of this, the charge density profile remains very much like that of the original plasma channel.

This relatively stable region of high charge density translates to a stable region of low resistance to the flow of electrons. Looking at the situation physically, future breakdown streamers will be enticed to follow this highly conductive path and either complete the breakdown process if it has not already occurred, or they will sustain the breakdown. This stable channel will permit breakdown to occur at lower current levels once the channel has been established.

Figure 9 is at the other end of the diffusion spectrum, that of a turbulent gas flow. Here, the diffusion coefficient is several orders of magnitude larger than in the laminar flow regime. The associated graph clearly demonstrates the rapid increase in the plasma channel's cross-section with time. This growth acts to reduce the overall charge

density within the plasma channel. Future streamers will see a higher path resistance than in the laminar flow case. If the path were created by a streamer which did not cause breakdown, future streamers will be less likely to follow the path and complete the breakdown process. If the plasma channel is a result of the breakdown process, future streamers will need to be more energetic to sustain the breakdown if they follow the original path.

As a result of the ionization process and the conclusions derived from the analysis of the 'governing equation', it is hypothesized that turbulent flow of the gaseous medium in a glow discharge process will delay the glow discharge to arc transition when compared to a laminar flow of gas.

VIII. CONCLUSIONS

This thesis presents the background concerning the process of de-ionization by two-body recombination and ambipolar diffusion in a gas. From this background, a description of the system is developed and analyzed. The results from this analysis demonstrate the effects of diffusion when the flow is either laminar or turbulent.

These results show that with the increase in turbulence and thus with the increase in the diffusion coefficient the charge density profile spreads out rapidly. This radial expansion reduces the charge density within the streamer discharge "tube". The results are somewhat inconclusive, however, when considering the physical process which takes place afterwards. To explain more clearly the question at hand, an examination of the post-streamer events in two limiting cases must first be examined.

First, assume the pure laminar flow situation whether there is negligible diffusion. Here, the streamer will retain its initial charge density profile as the charge carriers are lost by recombination. The streamer will not grow in diameter, but the charge density will decay more slowly than when diffusion is added. This streamer "tube" will maintain its high conductivity as the conductivity of a partially ion-ized plasma is proportional to the charge density.

The second case deals with a highly turbulent flow where there exist very large diffusion effects. The streamer in this situation will have

a rapidly expanding charge density profile. The charge carriers within the streamer will still be lost through recombination and the charge density will be further reduced through diffusion.

In the limit of numerous streamer events, the second case will establish a more uniform distribution of charge density throughout the gas. However, the first case of no diffusion will have numerous streamer "tubes" propagating through the gas, retaining their original charge density profile and decaying more slowly due to recombination.

Now, the question of follow-on streamers may be examined. Will the later streamers seek out the narrow, highly conductive channels in which to propagate further or will they propagate independently until the charge density within the gas as a whole is large enough to sustain a breakdown transition? Intuitively, either case may be argued successfully to some extent. Further research, more than likely experimental, is necessary.

In addition to the follow-on streamer problem, another interesting process might be worthy of consideration. Given the case of near turbulent flow, where diffusion acts to change the charge density profile, what takes place at relatively long time periods within the expanding streamer "tube"? Remember that the recombination rate is proportional to the square of the charge density (the number of ions times the number of free electrons). If the recombination coefficient is constant throughout the streamer "tube", areas of high density will decay faster than areas of lower density (as in the diffusion expansion region). This process could indicate the existence of a perturbation in

the charge density profile where the charge density is either less inside this location and radially outward or greater. The result would be some type of higher or lower density region (saddle point) at a given radial position within the charge density profile.

If this process is correct for times on the order of 10^{-5} to 10^{-4} seconds, then there may be an optimum level of diffusion which delays the process of streamer to arc breakdown or enhances the transition.

These questions remain unresolved at this point. Further research into may lead to an optimum level of turbulent mixing within the gaseous medium to maximize the ionization level within the gas and yet not allow the glow discharge to arc transition. This would maximize either the power output or the safety characteristics of electrically pumped lasers.

LIST OF REFERENCES

1. Dzakowic, S. and Wutzke, B., "High Pulse Rate Glow Discharge Stabilization by Gas Flow", Journal of Applied Physics, Vol. 16, #11, November 1973.

2. Ziganshin, Ismagilov and Minushev, "Effect of Turbulence on the Characteristics of a Glow Discharge", Journal of Engineering Physics, Vol. 39, #4, October 1980.

3. Nasser, E., Fundamentals of Gaseous Ionization and Plasma Electronics, Wiley, New York, p. 436, 1971.

4. McDaniel, E. W., Collision Phenomena in Ionized Gases, Wiley, New York, 1964.

5. Op. Cit., Nasser, p. 254.

6. Ibid., p. 265.

7. Ibid., p. 272.

8. Ibid., p. 273.

9. Ibid., p. 275.

10. Ibid., p. 330.

11. Ibid., p. 309.

12. Schwirzke, F., Unpublished Manuscript, Naval Postgraduate School, Monterey, California, 1982.

13. Op. Cit., Nasser, p. 323.

14. Ibid., p. 327.

15. Ibid., p. 340.

16. Chen, F., Introduction to Plasma Physics, Plenum Press, New York, 1974.

17. Del Croix, J. L., Introduction to the Theory of Ionized Gases, Interscience Pub. Inc., New York, 1960.

18. Op. Cit., Nasser, p. 377.

19. Ibid., p. 393.

20. Op. Cit., McDaniel, p. 563.

21. Ibid., p. 565.

22. Op. Cit., Chen, p. 146.

23. Biondi, M. and Chanin, L., "Mobilities of Atomic and Molecular Ions in Noble Gases", Physics Review, 94, p. 910, 1954.

24. Gray, E., Kerr, D., Physics Annual, New York, 17, 276, 1962.

25. Op. Cit., McDaniel, p. 612.

26. Ibid., p. 604.

27. Ibid., p. 564.

28. Loeb, L., "Theory of Recombination of Gaseous Ions", Physics Review, Vol. 51, p. 63, 1110, 1937.

29. Bondi, M., "Atomic Collisions Involving Low Energy Electrons and Ions", Advances in Electronics and Electronic Physics, Vol. 18, p 47, Academic Press, New York, 1963.

30. Douglas, A. and Hamilton, R., "Recombination Rate Measurements in Nitrogen", Journal of Chemical Physics, Vol. 58, #11, p. 108, June 1973.

31. Mitchner, Kruger, Partially Ionized Gases, Wiley, New York, 1973.

32. Ketter, Prawel, Modern Methods of Engineering Computation, McGraw Hill, New York, 1969.

33. Ibid., p. 172.

34. Ibid., p. 223.

35. Schwartz, Lavie, "Effects of Turbulence on a Weakly Ionized Plasma Column", AIAA Journal, Vol. 13, p. 22, May 1975.

36. Kestin, J., and Maeder, P., "The Influence of Turbulence on the Transfer of Heat to a Cylinder Near the Stagnation Point", ZMAP, Vol. 12, p. 115, 1961.

37. Schlichting, H., Boundary Layer Theory, McGraw Hill, New York, 1968.

38. Francis G., Ionization Phenomena in Gases, Academic, New York, 1960.

39. Bondi, M., "Concerning the Mechanism of Electron-Ion Recombination", Physics Review, Vol. 129, p. 1181, 1963.

BIBLIOGRAPHY

Biblarz, Dolson and Short, "Anode Phenomena in a Collision Dominated Plasma", Journal of Applied Physics, Vol. 46, #8, August 1975.

Douglas and Hamilton, "Recombination Rate Measurements in Nitrogen", Journal of Chemical Physics, Vol. 58, #11, June 1973.

Dzakowic and Wutzke, "High Pulse Rate Glow Discharge Stabilization by Gas Flow", Journal of Applied Physics, Vol. 44, #11, November 1973.

Grabec and Mikac, "Ionization Turbulence", Plasma Physics, Vol. 16, #12, December 1974.

Myshenkov and Makhviladze, "Suppression of the Ionization Instability in a Glow Discharge by Turbulent Transport", Soviet Journal of Plasma Physics, Vol. 4, #2, March 1978.

Naumov and Shashkov, "Combination Discharges in a Supersonic Gas Flow", Soviet Technical Letters, Vol. 13, November 1977.

Schwartz and Lavie, "Effects of Turbulence on a Weakly Ionized Plasma Column", AIAA Journal, Vol. 13, #5, May 1975.

Schwartz and Wasserstrom, "The Role of Gas Flow and Turbulence in Electric Discharge Lasers", Israel Journal of Technology, Vol. 13, 1975.

Wainionpaa, J., Electric Discharge Flow Interaction in Parallel and Cross-Flow Electric Fields, Master's Thesis, Naval Postgraduate School, September 1981.

Wasserstrom, Crispin, Rom, and Shwartz, "The Interaction Between Electric Discharges and Gas Flow", Journal of Applied Physics, Vol. 49, #1, January 1978.

Zhukov, Zasypkin, Mishne and Sazonov, "Electrical Field Intensity Due to an Arc in a Developed Turbulent Air Stream", Journal of Applied Mechanics and Technical Physics, Vol. 20, #6, November 1979.

Ziganshin, Ismagilov and Minushev, "Effect of Turbulence on the Characteristics of a Glow Discharge", Journal of Engineering Physics, Vol. 39, #4, October 1980.

APPENDIX A

THREE-BODY LINEAR SOLUTION

The following is the mathematical derivation of the three-body governing equation linearization process. Results are given in terms of the Gaussian Error Function (erf) for the analytical solution of the two-dimensional case.

The non-linear equation which describes the continuity equation for charge density in the cartesian coordinate system is:

$$\frac{\partial n}{\partial t} - D_a \frac{\partial^2 n}{\partial x^2} = -\alpha_2 n^2 \quad \text{for two-body} \tag{A1}$$

$$\frac{\partial n}{\partial t} - D_a \frac{\partial^2 n}{\partial x^2} = -\alpha_3 n^3 \quad \text{for three-body} \tag{A2}$$

The assumptions required are:

(1) Diffusion is a perturbation on the recombination controlled process

(2) $\tau_R << \tau_D$

Three-body linearized equation derivation:

$$\frac{\partial n}{\partial t} - D_a \frac{\partial^2 n}{\partial x^2} = -\alpha n^3 \qquad \text{(A3)}$$

$$\frac{\partial n}{\partial t} = -\alpha n^3 \qquad \text{(A4)}$$

yields: $$n = \left[\frac{n_o^2}{1+\alpha n_o t\, 2}\right]^{1/2} \qquad \text{(A5)}$$

let: $$n(x,t) = \tilde{n}(t) + \varepsilon\, n_1(x,t) \quad \text{where} \quad \varepsilon \ll 1 \qquad \text{(A6)}$$

$$\frac{\partial \tilde{n}}{\partial t} + \varepsilon \frac{\partial n_1}{\partial t} - \varepsilon D_a \frac{\partial^2 n_1}{\partial x^2} = -\alpha\left[\tilde{n} + \varepsilon n_1\right]^3 \qquad \text{(A7)}$$

H.O.T.

$$= -\alpha\left[\tilde{n}^3 + 3\varepsilon n_1 \tilde{n}^2 + \cancel{3\varepsilon^2 n_1^2 \tilde{n}} + \cancel{\varepsilon^3 n_1^3}\right] \qquad \text{(A8)}$$

because of equation (A4):

$$\frac{\partial n_1}{\partial t} - D_a \frac{\partial^2 n_1}{\partial x^2} = -3\alpha \bar{n}^2 n_1 \tag{A9}$$

$$\frac{\partial n_1}{\partial t} - D_a \frac{\partial^2 n_1}{\partial x^2} = \frac{-3\alpha n_1 n_0^2}{1+2\alpha n_0^2 t} \tag{A10}$$

When the appropriate boundary conditions are included the specific solution is:

$$n(x,t) = \frac{n_0(1-\varepsilon)}{(1+2\alpha n_0^2 t)^{1/2}} + \frac{\varepsilon n_0}{(1+\alpha n_0^2 t)^{3/2}}\left[\mathrm{erf}\left(\frac{x_0+x}{2\sqrt{D_a t}}\right) + \mathrm{erf}\left(\frac{x_0-x}{2\sqrt{D_a t}}\right)\right]; \; x \le x_0 \tag{A11}$$

$$= \frac{\varepsilon n_0}{(1+\alpha n_0^2 t)^{3/2}}\left[\mathrm{erf}\left(\frac{x_0+x}{2\sqrt{D_a t}}\right) + \mathrm{erf}\left(\frac{x_0-x}{2\sqrt{D_a t}}\right)\right]; \; x > x_0$$

APPENDIX B
NON-LINEAR PROGRAM

The program listing shows several features which need amplification. The first pertains to the centreline condition and how it effects the computational molecule. One boundary condition requires the slope of the density profile be zero at the centreline. This is used in the program to evaluate the charge density at the station next to the centreline. This value is the same as the centreline value. Due to this technique, the computational molecule does not need to "reach across" the centreline to find the (0-1,j) magnitude. The program uses the value at (0,j) and at (2,j) to evaluate the density at (1,j+1).

Figures 4 through 9 in Chapter VII are plotted to five times the initial streamer radius. However, the program calculates the charge density profile out to nineteen times the initial radius for each computation. The region not shown in the figures may be plotted by suitable changes to the plot commands in the program.

In the program, the diffusion coeffi cient (D_a) is based on the recombination time and the factor β. As β changes from laminar to turbulent conditions, D_a is changed accordingly. This technique was utilized to quickly adjust the flow conditions in the program.

As a test of the stability condition shown in equation (21), the following worst case situation is presented. The largest value of D_a is governed by the choice of β, as the other parameters are constant during iteration.

$$D_a = \frac{\alpha n_0 r_0^2}{\beta} \tag{B1}$$

Substituting the smallest ue; β = 50 yields a D_a of:

$$\frac{\alpha n_0 r_0^2}{\beta} = 5\times10^{-3} \tag{B2}$$

This is the largest value of D_a possible in this program. If this maximum D_a is substituted into equation (21) we find:

$$\Delta r^2 > 2 D_a \Delta t \tag{B3}$$

$$2.5\times10^{-11} > 2\times10^{-11} \tag{B4}$$

This confirms the assumed stability of the solution technique for values of $\beta > 50$. One could reverse the process and find the smallest value for β which satisfies the stability criteria if necessary.

FILE: NONLIN FORTRAN A1 NAVAL POSTGRADUATE SCHOOL

```
//WALLACE JOB (2246,0096),'WALLACE,R,J'CLASS=A
//EXEC FRTXCLGP
//FORT.SYSIN   DD  *
C
C
C        THE NON-LINEAR SOLUTION FOR NEAR TURBULENT FLOW
C
C
      REAL RO,NO,DA,ALFA1,ALFA2,DELTAT,DELTAR,N(199,199),A,B,C,INT,TIME
      REAL VAL(199),RR(199)
      INTEGER I,J,K,L,COUNT,KEY
C
C
C         INITIALIZE THE CONSTANTS
C
C
      RO=50.E-6
      NO=1.E20
      ALFA1=1.E-12
      ALFA2=1.E-34
      DELTAT=1./(alfa1*NO*10.)
      DELTAR=RO/10.
C
C
C        CALCULATE THE DIFFUSION COEFFICIENT BASED ON RECOMBINATION
C                   TIME AND BETA
C
C
      DA=(ALFA1*NO*RO*RO)/50.
C
C
C        IN THE ABOVE EQUATION, DIVIDE BY 5000 FOR LAMINAR FLOW
C               AND BY 50 FOR TURBULENT FLOW
C
C
C        SET THE MATRIX TO VIRTUAL ZERO
C
C
      DO 100 J=1,199
         DO 200 I=1,199
            N(I,J)=1.
  200    CONTINUE
  100 CONTINUE
      DO 10 I=1,11
         N(I,1)=NO
   10 CONTINUE
      DO 300 J=1,199
          TIME=DELTAT*J
```

```
      DO 400 I=2,191
         R=(I-1)*RO/10.
         A=(N((I+1),J)-(2.*N(I,J))+N((I-1),J))/(DELTAR*DELTAR)
         B=(N((I+1),J)-N((I-1),J))/(R*2.*DELTAR)
         C=(A*(A+B)-(ALFA1*N(I,J)*N(I,J))-(ALFA2*N(I,J)**3.)
         INT=R/RO
         IF (I.NE.2) GO TO 111
            N((I-1),(J+1))=N(I,(J+1))
  111    CONTINUE
  400 CONTINUE
  300 CONTINUE
C
C        TRANSFER  THE RESULTS FOR THE PLOT ROUTINE
C
      DO 8478 J=1,191
            DO 8477 I=1,50
                  VAL(I)=(LOG(N(I,J)+1.))/2.303
                  RR(I)=(I-1)/10.
 8477       CONTINUE
C
C        VERSATEK PLOT ROUTINE
C
 8478 CONTINUE
      CALL PLOTG(RR,VAL,50,L,1,L,'R/RO',4,"LOG(N)',6,0,0,0,0,6,6)
      CALL PLOT(0.0,0.0,999)
  999 FORMAT(' ',29X,F6.2,10X,E10.3)
  888 FORMAT(' ')
  555 FORMAT(' ',E9.2,5X,E9.2)
      STOP
      END
/*
//GO.SYSIN  DD  *
/*
```

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3.	Office of Research Administration, Code 012A Naval Postgraduate School Monterey, California 93940	2
4.	Chairman Department of Aeronautics, Code 67 Naval Postgraduate School Monterey, California 93940	1
5.	Professor Oscar Biblarz Department of Aeronautics, Code 67Bi Naval Postgraduate School Monterey, California 93940	5
6.	LT. Richard J. Wallace, USN #26 La Playa Monterey, California 93940	2
7.	Professor F. Schwirzke Department of Physics, Code 61Sw Naval Postgraduate School Monterey, California 93940	1
8.	LCDR J. L. Barto, USN Weapons Systems Officer, Code 2004 Pacific Missile Test Centre Point Magu, California 93042	1
9.	Dr. J. Shwartz TRW System One Space Park Redondo Beach, California 90278	1

10. Dr. Alan Garscadden 1
AFAPL/POD
Building 450/Room D101
Wright-Patterson AFB, Ohio 45433

11. John F. Wallace 1
IBM Corporation
7226 Old Gate Road
Rockville, Maryland 20852

END

DATE
FILMED

6-83

DTIC